

Strings with interacting ends^{*}

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Abstract

At the classical level we study open bosonic strings. A generic description of string self-interactions localized at string ends is given. Self-interactions are characterized by two dimensionless coupling constants. The model is rewritten using complex Liouville fields. Using these Lorentz and reparametrization invariant variables, equations of motion get greatly simplified and reduce to some boundary problem for Liouville equation.

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The classical dynamics of relativistic strings follows entirely from the minimal action principle. The action integral is some reparametrization invariant functional of time-like two-dimensional surfaces. For open strings, equations of motion derived from the action consist of two groups, bulk and boundary equations. Bulk equations of motion are Euler–Lagrange variational equations. In the simplest case of non-interacting Nambu–Goto strings they are Laplace–Beltrami equations and their solutions represent minimal surfaces, i.e. (time-like) surfaces of zero mean external curvature. For open Nambu–Goto strings, we know that Laplace–Beltrami equations should be supplemented by (von Neumann type) boundary conditions for world sheet coordinates. In general, if we refer to any would-be string model, boundary conditions to be satisfied at open string ends have a form of dynamical equations. They can describe some kind of string self-interactions. In the paper, we want to follow this point through. We believe that the problem is essential while considering string models of hadrons. The description of the hadronic string should have regard to the presence of quarks at the ends, so that it is instructive to examine possible interactions between the string and its ends.

Let us begin with the introduction of the generic form of the action functional for open strings.

$$S = \int_{\tau_1}^{\tau_2} d\tau \int_{\sigma_1(\tau)}^{\sigma_2(\tau)} d\sigma \sqrt{-g} \mathcal{L} + \int_{\tau_1}^{\tau_2} d\tau L^{(1)} + \int_{\tau_1}^{\tau_2} d\tau L^{(2)} , \quad (1)$$

where \mathcal{L} is a scalar function (with respect to both Poincare and reparametrization transformations) of string coordinates. It can be always presented as a functional of induced metric $g_{ab} = X_{,a}^\mu X_{\mu,b}$ and defined with its help covariant derivatives of world sheet coordinates $X_\mu(\sigma^a)$ ($\sigma^a = (\tau, \sigma)$):

$$\mathcal{L} = \mathcal{L}(g^{ab}; \nabla_a X_\mu; \nabla_a \nabla_b X_\mu; \dots) , \quad (2)$$

$$\nabla_a X_\mu \equiv \frac{\partial X_\mu}{\partial \sigma^a} \equiv X_{\mu,a} , \quad \nabla_a \nabla_b X_\mu \equiv X_{\mu,ab} - \Gamma_{ab}^c X_{\mu,c} .$$

The dots stand for higher covariant derivatives, additional fields placed on the world sheet or external fields coupled with strings. Lagrangians $L^{(i)}$ are functionals of string ends trajectories and their total time derivatives:

$$L^{(i)} = L^{(i)}(X_\mu; d_t X_\mu; \dots)|_{\sigma=\sigma_i(\tau)} , \quad (3)$$

$$d_t = \nabla_0 + \dot{\sigma}_i \nabla_1 .$$

Similarly, the dots stand here for higher total time derivatives and couplings with external fields. Poincare-invariant functionals $L^{(i)}$ should be also scalar densities with respect to the change of parametrization of string ends trajectories (such transformations can be a part of any total world sheet reparametrization).

The bulk classical equations of motion derived from the action integral (1) follow entirely from the first action term. They can be written down in a manifestly covariant form [1]:

$$\sqrt{-g} \nabla_a \Pi_\mu^a = 0 , \quad (4)$$

where Π_μ^a is given by the formula

$$\begin{aligned} \Pi_\mu^a = & -\mathcal{L} \nabla^a X_\mu - \frac{\partial \mathcal{L}}{\partial X_{,a}^\mu} + 2 \frac{\partial \mathcal{L}}{\partial g^{bc}} g^{ab} \nabla^c X_\mu \\ & + \nabla_b \left[\frac{\partial \mathcal{L}}{\partial (\nabla_a \nabla_b X^\mu)} \right] . \end{aligned} \quad (5)$$

From now, we restrict our discussion to string models defined by Lagrangians that depend on no higher than second order derivatives. The above formula is much more simpler to evaluate Euler-Lagrange equations than the standard one which includes variational derivatives taken with respect to non-covariant world sheet derivatives. As usual, performing variational derivatives we regard g^{01} and g^{10} , $\nabla_0 \nabla_1 X_\mu$ and $\nabla_1 \nabla_0 X_\mu$ as independent variables. Then, all variational derivatives are tensor objects.

For open strings, the variational problem results also in 'the edge conditions', being in fact dynamical equations of motion to be held at world sheet boundaries (trajectories of string end points). The boundary equations of motion are collected below

$$\begin{aligned} & \sqrt{-g} \Pi_\mu^1 + \partial_0 \left[\sqrt{-g} \frac{\partial \mathcal{L}}{\partial (\nabla_0 \nabla_1 X^\mu)} \right] - \partial_1 \left[\sqrt{-g} \frac{\partial \mathcal{L}}{\partial (\nabla_0 \nabla_0 X^\mu)} \right] \dot{\sigma}_i^2 \\ & - \left\{ \sqrt{-g} \Pi_\mu^0 + \partial_0 \left[\sqrt{-g} \frac{\partial \mathcal{L}}{\partial (\nabla_0 \nabla_0 X^\mu)} \right] - \partial_1 \left[\sqrt{-g} \frac{\partial \mathcal{L}}{\partial (\nabla_0 \nabla_1 X^\mu)} \right] \right\} \dot{\sigma}_i \\ & - \sqrt{-g} \frac{\partial \mathcal{L}}{\partial (\nabla_0 \nabla_0 X^\mu)} \ddot{\sigma}_i + (-1)^i d_t \left(\frac{\partial \mathcal{L}}{\partial d_t X^\mu} \right) - (-1)^i d_t^2 \left(\frac{\partial \mathcal{L}}{\partial d_t^2 X^\mu} \right) = 0 , \end{aligned} \quad (6)$$

$$\begin{aligned} \sqrt{-g} \frac{\partial \mathcal{L}}{\partial(\nabla_1 \nabla_1 X^\mu)} - \left[\sqrt{-g} \frac{\partial \mathcal{L}}{\partial(\nabla_0 \nabla_1 X^\mu)} + \sqrt{-g} \frac{\partial \mathcal{L}}{\partial(\nabla_1 \nabla_0 X^\mu)} \right] \dot{\sigma}_i \\ + \sqrt{-g} \frac{\partial \mathcal{L}}{\partial(\nabla_0 \nabla_0 X^\mu)} \dot{\sigma}_i^2 = 0 , \quad \text{for } \sigma = \sigma_i(\tau) . \end{aligned} \quad (7)$$

In the ordinary Nambu–Goto model \mathcal{L} is a constant and ‘point-like’ terms in (1) are absent. The Nambu–Goto string action can be considered as the action of a non-interacting string. Any kind of string self-interactions must involve higher derivatives of world sheet coordinates in the action. In this paper, we consider only such interactions that leave bulk equations of motion unchanged (i.e. the same as in the Nambu–Goto case):

$$g^{ab} \nabla_a \nabla_b X_\mu = 0 . \quad (8)$$

Thus, possible string self-interactions are localized at string ends. A general form of the action that results in the Nambu–Goto bulk string equations has been derived in [1]:

$$\mathcal{L} = -\gamma - \frac{\alpha}{2} R - \beta N , \quad (9)$$

where γ is string tension, α and β are dimensionless parameters. The first constant term in (9) corresponds to the Nambu–Goto action, while other terms are respectively the integrands of Gauss–Bonnet and Chern invariants for two-dimensional surfaces. Scalar functions R and N have the form

$$R = (g^{ab} g^{cd} - g^{ad} g^{bc}) (\nabla_a \nabla_b X_\mu) \nabla_c \nabla_d X^\mu , \quad (10)$$

$$N = -\frac{1}{2\sqrt{-g}} g^{ac} \epsilon^{bd} \tilde{t}^{\mu\nu} (\nabla_a \nabla_b X_\mu) \nabla_c \nabla_d X_\nu , \quad (11)$$

where $\tilde{t}^{\mu\nu} = \frac{1}{\sqrt{-g}} \epsilon^{\mu\nu\rho\sigma} X_{\rho,0} X_{\sigma,1}$.

For point-like terms in the action (1), we restrict ourselves to the simplest choice and take invariant lengths of trajectories of string ends [2–4], namely

$$L^{(i)} = -m_i \sqrt{(d_t X)^2} \Big|_{\sigma=\sigma_i(\tau)} . \quad (12)$$

Physically, we can say that point-like masses m_1 and m_2 have been attached to string ends.

A great technical advantage of working with Nambu–Goto strings is that bulk equations of motion (8) can be linearized by a suit choice of gauge. Practically, any kind of string self-interactions that changes bulk equations (8) complicates them in such a drastic way that makes the model mathematically hardly tractable (we have to deal with a non-linear system of partial differential equations of fourth order in time and space derivatives). The purpose of this paper is to establish a suitable formalism to examine string self-interactions that result merely in boundary equations of motion. It is astonishing that if we try to construct invariant action terms for this type of self-interactions, then we find only two possibilities (displayed in (9)). To prove this fact (see Ref.[1]) we need only consider requirements of Poincare and reparametrization invariances. It is important to stress that in this proof *no* assumptions are made on analytical form of invariant terms. This makes our statement really strong. For example, let us compare it with another well-known statement that scalar functions R , N and a square of Laplace–Beltrami operator (Polyakov rigidity term) are the only invariants we can construct for two-dimensional surfaces immersed in four-dimensional flat spacetime. This statement is true as long as we restrict ourselves to action terms for surfaces such that their integrands are polynomials in external curvature tensor coefficients and respective coupling constants have “renormalizable” dimensions. If we weaken any of these assumptions, we find immediately infinitely many new candidates for action terms.

Let us start our treatment of the model defined by the action (1), with Lagrangians specified by (9) and (12). As it has been explained, string Lagrangian (9) is the most general form allowed by a restriction that the variational problem results in Nambu–Goto bulk equations of motion (8), and point-like Lagrangian (12) is just the simplest choice. However, it is the only possible choice unless we let higher than third order derivatives appear in boundary equations. Boundary equations of motion (6–7) arise from string self-interaction terms in (9) and point-like Lagrangians (12). These equations are pretty complicated non-linear differential equations with third derivatives in time and space parameters if they are expressed in terms of world sheet coordinates X_μ . They are to be held at both string ends $\sigma = \sigma_i$ at any time.

We will make use of the following convenient way of parametrizing

string world sheets [5]

$$\begin{aligned}(X_{\mu,0} \pm X_{\mu,1})^2 &= 0 , \\ (X_{,00} \pm X_{,01})^2 &= -q^2 ,\end{aligned}\tag{13}$$

where q is some arbitrary constant of mass dimension. The first set of parametrization conditions (orthonormal gauge) makes bulk equations of motion (8) linear and their general solution can be represented as the combination of left- and right-moving parts,

$$X_\mu(\tau, \sigma) = X_{L\mu}(\tau + \sigma) + X_{R\mu}(\tau - \sigma) .\tag{14}$$

As the orthonormal gauge still allows for conformal changes of parametrization, so that the latter pair of parametrization conditions in (13) is chosen to kill completely this residual gauge freedom. Note that this supplementary gauge is possible only for world sheets obeying bulk equations of motion.

Up to transformations of Poincare group, minimal surfaces X_μ (i.e. surfaces satisfying bulk equations (8) parametrized according to (13)) correspond to solutions Φ of the complex Liouville equation [1]:

$$\ddot{\Phi} - \Phi'' = 2q^2 e^\Phi .\tag{15}$$

The real part of Liouville field $\text{Re}\Phi$ is the only independent part of the induced metric in the orthonormal gauge:

$$\sqrt{-g} = e^{-\text{Re}\Phi} .\tag{16}$$

While $\text{Re}\Phi$ describes fully internal geometry of the string world sheet, the imaginary part of Liouville field $\text{Im}\Phi$ characterizes its extrinsic geometry, i.e. the way world sheet is embedded in four-dimensional spacetime. In terms of geometrical objects, we can describe this embedding by extrinsic torsion coefficients:

$$\omega_a = n^{1\mu} \partial_a n_\mu^2$$

where $n_\mu^1(\tau, \sigma)$ and $n_\mu^2(\tau, \sigma)$ are vectors normal to the world sheet at point (τ, σ) .

$$n^{i\mu} X_{\mu,a} = 0 , \quad n^{i\mu} n_\mu^j = -\delta^{ij} .\tag{17}$$

Normal vectors n_μ^i are defined modulo their common rotation in a local plane perpendicular to the world sheet. Such local rotation about some angle $\phi(\tau, \sigma)$ changes torsion coefficients: $\omega_a \rightarrow \omega_a + \partial_a \phi$. Therefore, the only meaningful characteristics of extrinsic geometry is $N = -\frac{1}{\sqrt{-g}} \epsilon^{ab} \partial_a \omega_b$. One can convince oneself [1] that it coincides with defined previously scalar function (11) and has the following relation with the imaginary part of Liouville field:

$$N = \frac{q^2}{g} \sin(\text{Im}\Phi) . \quad (18)$$

For practical purposes, it is most convenient to express the correspondence between world sheet coordinates and complex Liouville field in the following way:

$$\begin{aligned} e^\Phi &= -\frac{4}{q^2} \frac{f'_L(\tau + \sigma) f'_R(\tau - \sigma)}{[f_L(\tau + \sigma) - f_R(\tau - \sigma)]^2} , \\ \frac{\partial}{\partial \tau} X_{L,R}^\mu &= \frac{q}{4|f'_{L,R}|} (1 + |f_{L,R}|^2, 2\text{Re } f_{L,R}, 2\text{Im } f_{L,R}, 1 - |f_{L,R}|^2) , \end{aligned} \quad (19)$$

where f_L and f_R are arbitrary complex functions. The former formulae is a general solution of Liouville equation (15), the latter is a general solution of Nambu–Goto equations (8) satisfying gauge conditions (13). Arbitrary functions f_L and f_R in both formulae can be identified iff we put (16) and (18). Note that any simultaneous modular transformation of f_L and f_R induces Lorentz transformation of X_μ while Liouville field Φ remains invariant.

In this paper, our main purpose will be to express bulk and boundary equations of motion for an open string, that follow from the action (1) with Lagrangians (9) and (12), in terms of Liouville variables.

As it was stated, instead of bulk equations of motion (8) there appears complex Liouville equation (15). Thus, our task is to rewrite boundary equations (6–7). If we insert in general formulae our Lagrangians and perform all calculations, then after using (8) and (13) we can present boundary equations at $\sigma = \sigma_i(\tau)$ in the form

$$\begin{aligned} &\gamma X_{\mu,1} + \tilde{Y}_{\mu,0} + (\gamma X_{\mu,0} + Y_{\mu,0} + \tilde{Y}_{\mu,1}) \dot{\sigma}_i \\ &+ Y_{\mu,1} \dot{\sigma}_i^2 + Y_\mu \ddot{\sigma}_i + (-1)^i m_i d_t \left(\frac{d_t X_\mu}{\sqrt{(d_t X)^2}} \right) = 0 , \end{aligned} \quad (20)$$

$$Y_\mu + 2\tilde{Y}_\mu \dot{\sigma}_i + Y_\mu \dot{\sigma}_i^2 = 0 , \quad (21)$$

where we have introduced:

$$Y_\mu \equiv \frac{\alpha}{\sqrt{-g}} \nabla_0 \nabla_0 X_\mu + \frac{\beta}{\sqrt{-g}} \tilde{t}_{\mu\nu} \nabla_0 \nabla_1 X^\nu ,$$

$$\tilde{Y}_\mu \equiv \frac{\alpha}{\sqrt{-g}} \nabla_0 \nabla_1 X_\mu + \frac{\beta}{\sqrt{-g}} \tilde{t}_{\mu\nu} \nabla_0 \nabla_0 X^\nu .$$

Taking into account definitions of Y_μ , \tilde{Y}_μ , R , N and gauge conditions (13) one can derive the identities:

$$Y^\mu X_{\mu,a} = \tilde{Y}^\mu X_{\mu,a} = 0 ,$$

$$Y^\mu X_{\mu,01} = \tilde{Y}^\mu X_{\mu,00} = 0 ,$$

$$Y^\mu X_{\mu,00} = -\frac{\alpha q^2}{2\sqrt{-g}} - \frac{\alpha}{4} \sqrt{-g} R - \frac{\beta}{2} \sqrt{-g} N ,$$

$$\tilde{Y}^\mu X_{\mu,01} = -\frac{\alpha q^2}{2\sqrt{-g}} + \frac{\alpha}{4} \sqrt{-g} R + \frac{\beta}{2} \sqrt{-g} N . \quad (22)$$

Now, one can easily deduce from Eq.(21) that

$$(1 + \dot{\sigma}_i^2) Y X_{,00} = \dot{\sigma}_i \tilde{Y} X_{,01} = 0 . \quad (23)$$

Combining the above equations with identities (22) one can obtain the following requirement:

$$\dot{\sigma}_i = 0 , \quad i = 1, 2 . \quad (24)$$

It is a great technical advantage of working with gauge conditions (13) that we can put in (1) space parameter interval independent of time. Here, it is a consequence of equations of motion. Of course, it was possible at the beginning, as it is usually done, to fix say $\sigma_1 = 0$ and $\sigma_2 = \pi$ by a proper reparametrization. But, it restricts the set of allowed parametrizations and we were not sure whether further parametrization conditions, like (13), would be acceptable. Open rigid strings provide us with an example that it may happen for some configurations that there exists no parametrization which both makes σ -interval time-independent and fulfills orthonormality conditions $(X_{,0} \pm X_{,1})^2 = 0$ (see [6]).

Now, and for the remainder of this paper, we will assume

$$\sigma_1 = 0, \sigma_2 = \pi. \quad (25)$$

Note that gauge conditions (13) fix world sheet parametrization almost uniquely, allowing only for constant shifts of τ and σ parameters. Therefore, passing from (24) to (25) we cannot fix the length of σ -interval to be π for all open string configurations. It means that this constant length is an integral of motion (related to scaling symmetry). As parameter q specified in (13) was an arbitrary constant and there is an obvious interplay between q and $\sigma_2 - \sigma_1$, we choose instead that (25) is fixed while q will be regarded from now as an integral of motion.

Taking into account (25), boundary equations of motion (20–21) at string ends $\sigma = 0, \pi$ reduce to

$$\gamma X_{\mu,1} + \tilde{Y}_{\mu,0} + (-1)^i m_i \frac{\partial}{\partial \tau} \left(\frac{X_{\mu,0}}{\sqrt[4]{-g}} \right) = 0, \quad (26)$$

$$Y_\mu = 0. \quad (27)$$

Finally, after specifying some orthonormal frame (17) we express covariant derivatives (normal to the world sheet) using imaginary part of Liouville field:

$$\nabla_0 \nabla_0 X_\mu = q \cos(\text{Im}\Phi/2) \epsilon^{ij} t^j n_\mu^i, \quad \nabla_0 \nabla_1 X_\mu = q \sin(\text{Im}\Phi/2) t^i n_\mu^i, \quad (28)$$

where $t^i(\tau, \sigma)$ are some arbitrary functions with no geometrical meaning, satisfying $t^i t^i = 1$. Then, the boundary equations of motion (26,27) at $\sigma = 0, \pi$ can be expressed in terms of Liouville field:

$$\gamma - \alpha q^2 e^{2\text{Re}\Phi} = (-1)^i m_i \frac{\partial}{\partial \sigma} \left(e^{\text{Re}\Phi/2} \right), \quad (29)$$

$$C \frac{\partial}{\partial \tau} \text{Re}\Phi = 0, \quad (30)$$

$$C \cos(\text{Im}\Phi/2) = \beta, \quad (31)$$

$$C \frac{\partial}{\partial \sigma} \text{Im}\Phi = (-1)^i 2m_i e^{-\text{Re}\Phi/2} \cos(\text{Im}\Phi/2), \quad (32)$$

where $C = \sqrt{\alpha^2 + \beta^2}$. From (30) and (31) follow that Liouville field Φ is constant and finite (for $C \neq 0$) at boundary points.

It is seen that the analysis of classical equations of motion has been greatly simplified due to the introduction of Liouville fields. We end up with some boundary problem (29–32) for Liouville equation (15). Let us make a short account of the most important points in our construction. We have elaborated a classical model of open strings in four-dimensional flat spacetime. The model is defined by the action (1) with Lagrangians (9) and (12). It is the most general string model as long as (i) string self-interactions are assumed to appear only in boundary equations of motion (ii) equations of motion contain no higher than third derivatives (iii) no additional internal fields on the world sheet or external fields in the target space are present. The generic string action depends on four parameters, two dimensionless coupling constants and two masses. The variational problem for open strings results in usual Nambu–Goto bulk equations and boundary equations, being a set of third order differential equations. Next, we define world sheet parametrization in a unique way (13) and introduce new independent variables (16) and (18), which are combined to a complex Liouville field. Equations of motion reduce to a boundary problem for complex Liouville equation.

Further analysis of the model of strings with interacting ends will be made in the following papers. Some special case has been considered in [7, 8].

In the last part of this paper, we discuss possible singularities of Liouville fields. An imaginary part of Φ can be interpreted as some angle variable, so that we are to consider singularities of $Re\Phi$. First, let us remind the relation between $Re\Phi$ and the determinant of the induced world sheet metric (16). If $Re\Phi < 0$ near some singular point, then it follows that invariant area of world sheet piece being a neighbourhood of this point is infinite. Under ordinary circumstances, such solutions are not physically acceptable. Thus, singular points can be taken into account provided that $Re\Phi > 0$ in their close vicinities.

Let us now look into the relations (19). We should consider four critical cases:

- (a) $|f_L|$ or $|f_R|$ goes to infinity
 - (b) $|f'_L|$ or $|f'_R|$ goes to infinity
 - (c) $f'_L = 0$ or $f'_R = 0$
 - (c) $f_L = f_R$
- at some point (τ_0, σ_0) .

First, it is helpful to make the following observation: $X_{L,0}^\mu$ or $X_{R,0}^\mu$ cannot

vanish. If $X_{L,0}^\mu = 0$ at some point (τ_0, σ_0) , then $X_{L,0}^\mu = 0$ at any point (τ, σ) such that $\tau + \sigma = \tau_0 + \sigma_0$. In particular, it would vanish at some boundary point. It is impossible as $e^{-\text{Re}\Phi} = 2X_{L,0}X_{R,0}$ and $\text{Re}\Phi$ is finite at boundary points. Because $X_{L,0}^\mu$ and $X_{R,0}^\mu$ are light-like vectors, it implies that their time components cannot vanish.

Keeping in mind the above observation, we can establish that cases (a) and (b) can be taken into account only if ratios f'_L/f_L^2 and f'_R/f_R^2 are finite and non-vanishing at (τ_0, σ_0) . It is straightforward to show further that Liouville field has no singularities at such points and we can easily get rid of (a) and (b) by performing a suitable modular transformation.

In case (c), suppose that $f'_L(\tau_0 + \sigma_0) = 0$. If $\text{Re}\Phi > 0$, then $f_L(\tau_0 + \sigma_0) = f_R(\tau - \sigma)$ at any point (τ, σ) such that $\tau_0 + \sigma_0 = \tau + \sigma$. It implies that f_R is a constant function in some finite interval, but we can easily convince ourselves that it is impossible. Thus, the case (c) cannot occur, i.e. first derivatives of f_L and f_R cannot take zero values.

Finally, the case (d) is the only one when we can admit a singularity of Liouville field Φ . If $f_L(\tau_0 + \sigma_0) = f_R(\tau_0 - \sigma_0)$, then $\sqrt{-g} = 0$ at (τ_0, σ_0) . Such string points travel with light-like velocities.

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